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The distortion of turbulence by irrotational plane strain

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The experiments reported here extend those of Townsend which form the basis of his model of free turbulence. Here straining is carried to a strain ratio of 6:1, while Townsend's straining went only to 4:1. Two kinds of distorting ducts are used to produce the uniform mean strain applied to initially nearly isotropic grid turbulence.

The results differ from Townsend's in that: (i) a considerably higher degree of anisotropy is achieved, Townsend's measure of anisotropy attaining values up to 0·6, rather than the maximum of 0·42 he found; (ii) there is no evidence that an equilibrium structure is attained; and (iii) the strained turbulence rapidly becomes less anisotropic when the straining ceases.

It is found to be possible to predict the variation of the total turbulence energy using rapid-distortion theory with a correction for decay. However, the individual components cannot be accurately predicted in this way.

1. Introduction

The relationship between mean strain and the structure of turbulence is of fundamental importance in all turbulent shear flows. However, in non-homogeneous strain fields (for example, a boundary layer) turbulent fluid is convected through a variety of strain patterns, making it difficult to relate the local mean characteristics of the turbulence to the history of strain. The only studies which give direct information about the strain-structure relationship are those of flows which impose a homogeneous mean strain, uniform over a considerable time, on turbulence which is itself homogeneous.

The first nearly homogeneous strain fields to be studied were those in the contractions of wind tunnels, one of whose functions is the suppression of the kinetic energy of turbulence relative to that of the stream. The turbulence at the beginning of strain is fairly homogeneous, since uniform screens are usually introduced just upstream of the contraction with the intention of smoothing the flow in the test section. A theoretical study of this problem was undertaken by Taylor (1935), using a model of turbulence which consisted of a three-dimen-

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sional array of cubical vortices. The effects of strain on this model were determined using Cauchy's equations embodying the assumption that circulation is conserved (see Lamb 1932, p. 205). The same approach was used by Batchelor & Proudman (1954) and independently by Ribner & Tucker (1952). However, these workers replaced Taylor's simple model by a Fourier integral representation of the random turbulent field, thus separating the turbulent motion into a summation of sinusoidal components. All these analyses neglected the influence of viscosity and the interactions between eddies. Since these simplifications would apparently be permissible for instantaneous or very rapid distortion the work is often referred to as the 'rapid distortion theory'.

Although the turbulence and strain in wind-tunnel contractions are fairly homogeneous, the strain field can be uniform only by accident, since the contractions are not designed to achieve this end. Experimental studies of the flow in ducts specially designed to establish uniform strain were undertaken by Taylor's co-workers, first MacPhail (1944) and later, in much greater detail, by Townsend (1954).

The results of Townsend's experiments are of special interest since they form the basis of his far-reaching attempt to describe the response of turbulence to straining, embracing a wide range of non-homogeneous and non-uniform flows. These extensions are based on the idea that, after a certain degree of straining, an equilibrium structure of the turbulence is established. The mechanisms which transfer energy amongst the components are then so effective that further strain will produce little alteration in the structure. The equilibrium turbulence with its limited measure of anisotropy is expected to exist in many kinds of shear flow. These ideas are developed in Townsend's (1956) book.

Those of Townsend's experimental results of particular interest are: (i) isotropic turbulence is converted to the equilibrium structure by the imposition of a plane strain ratio of 4:1, such that an element of fluid is changed from the shape of 1:4:1 to 1:1:4; (ii) the equilibrium structure is characterized by the anisotropy measure $K_1 = (\overline{v^2}-\overline{w^2})/(\overline{v^2}+\overline{w^2}) = 0{\cdot}42$, v and w are the velocity fluctuations in the y and z directions associated with strain ratios of 4 and $\frac{1}{4}$ respectively; (iii) when the straining ceases, on the passage of the fluid into a uniform duct following the distorting section, the rate of return to isotropy is very slow. This result was checked by Grant (1958).

These experimental findings have such great intrinsic interest, and the conclusions drawn from them are of such far-reaching importance, that it seemed worthwhile repeating Townsend's measurements. Certain alterations have been made in the experimental arrangements, the most important being the lengthening of the duct to allow a higher strain ratio and the use of two species of distorting duct to generate the same strain field. As will be seen later, the conclusions reached on the basis of this study are in marked disagreement with Townsend's findings.

Although the experimental investigation is the main topic of this paper, we begin with a sketch of the theory of response to straining. These results are then available for comparison with the experimental data. Details of experimental apparatus and procedures are relegated to an appendix.

2. Theory of response of turbulence to strain

2.1. *Rapid distortion theory*

In considering the effect of strain on a turbulent fluid it is preferable to use the vorticity equation obtained by taking the curl of the Navier–Stokes equation to eliminate the pressure terms. Let D/Dt denote the total derivative with respect to time t, ω the vorticity due to the velocity fluctuations, U the mean velocity,

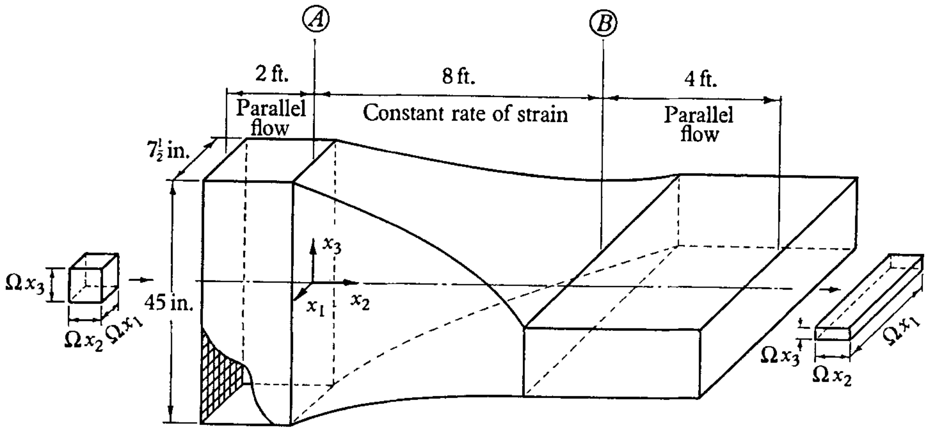

FIGURE 1. Laterally distorting tunnel, schematic representation of the section of wind tunnel producing a constant positive rate of strain in the horizontal direction and an equal negative rate of strain in the vertical direction.

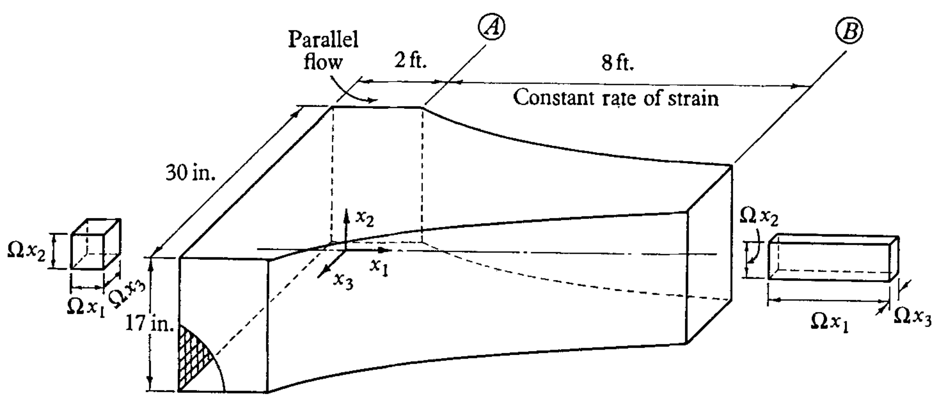

FIGURE 2. Longitudinally distorting tunnel, schematic representation of the section of wind tunnel producing a constant positive rate of strain in the longitudinal direction and an equal negative rate of strain in the horizontal direction.

u the fluctuating velocity and ν the kinematic viscosity. Using the usual tensor subscripts i and j to denote 1, 2, 3 the direction of the axes and summing over the repeated subscripts, the vorticity equation for an incompressible fluid with no mean rotation is (see Lamb 1932, p. 578)

$$\underset{\text{I}}{\frac{D\omega_i}{Dt}} = \underset{\text{II}}{\omega_j\frac{\partial U_i}{\partial x_j}} + \underset{\text{III}}{\omega_j\frac{\partial u_i}{\partial x_j}} + \underset{\text{IV}}{\nu\frac{\partial^2 \omega_i}{\partial x_j^2}}. \tag{1}$$

This equation states that (I) the rate of change in vorticity of the turbulent motion is equal to (II) the production of vorticity by the mean motion straining the turbulent eddies, plus (III) the transfer of vorticity amongst turbulent eddies by one eddy straining another, plus (IV) the diffusion of vorticity by viscosity. For an instantaneous or very rapid mean strain the transfer and viscous terms may be neglected. The vorticity equation then simplifies to

$$\frac{D\omega_i}{Dt} = \omega_j \frac{\partial U_i}{\partial x_j}. \tag{2}$$

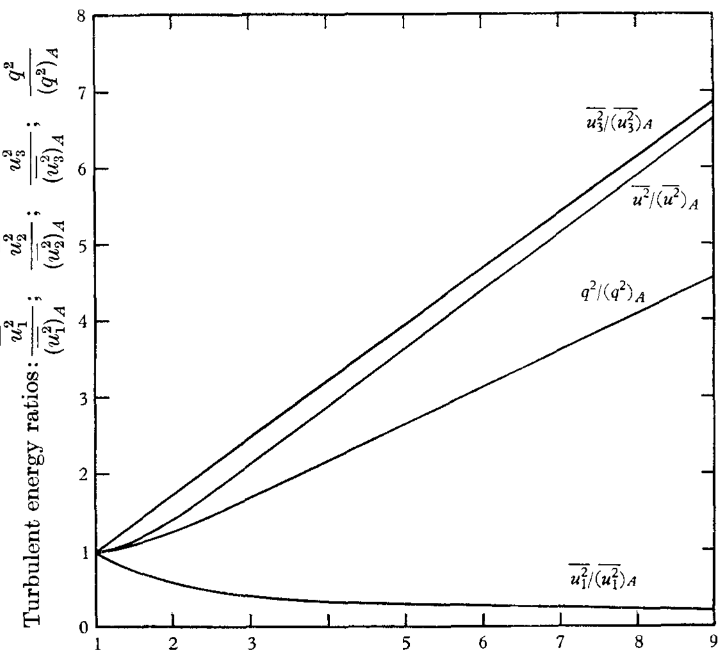

FIGURE 3. Changes in turbulent energy ratios for plane strain, calculated from the equations of Batchelor & Proudman (1954) for rapid distortion of initially isotropic turbulence.

Suppose that in passing through the mean strain field between stations A and B (figures 1 and 2), a line segment Ωx_i following the mean fluid motion has its length changed by a strain ratio l_i. Here and throughout the paper it was most convenient to choose reference axes related to the strain rather than the main flow direction. The x_1, x_2, and x_3 axes are chosen so that the strain ratios $l_1 \geqslant l_2 \geqslant l_3$. Subscripts A and B are used to denote conditions at sections A and B respectively. A consequence of equation (2) (see Lamb 1932, p. 206) is

$$\frac{(\omega_1)_B}{(\omega_1)_A} = \frac{(\Omega x_1)_B}{(\Omega x_1)_A} = l_1; \quad \frac{(\omega_2)_B}{(\omega_2)_A} = \frac{(\Omega x_2)_B}{(\Omega x_2)_A} = l_2; \quad \frac{(\omega_3)_B}{(\omega_3)_A} = \frac{(\Omega x_3)_B}{(\Omega x_3)_A} = l_3. \tag{3}$$

These equations are essentially those used by Taylor (1935). They relate the vorticity before and after strain and give the entire effect of instantaneous strain on turbulence.

Making use of (3), Batchelor & Proudman (1954) have found, theoretically, the effects of an arbitrary strain on turbulence. When the initial turbulence is isotropic, the turbulent kinetic energy associated with each component of velocity after strain is independent of the details of the initial spectrum. For the special case of a plane strain (where one of the three strain ratios remains unity) results are presented in a graphical form for strain ratios below 3. For greater strain ratios the following simplified formulae are given, the bar above denoting time mean values,

$$\mu_1 = \frac{\overline{u_1^2}}{(\overline{u_1^2})_A} = \frac{3}{4l_1}(\log 4l_1 - 1),$$

$$\mu_2 = \frac{\overline{u_2^2}}{(\overline{u_2^2})_A} = \tfrac{3}{4}l_1 - \frac{3}{8l_1}(\log 4l_1 - \tfrac{3}{2}),$$

$$\mu_3 = \frac{\overline{u_3^2}}{(\overline{u_3^2})_A} = \tfrac{3}{4}l_1 + \frac{3}{8l_1}(\log 4l_1 - \tfrac{1}{2}).$$

These results of Batchelor & Proudman have been used to obtain figure 3 of this paper, and permit a comparison of the present experimental results with those of the rapid distortion theory.

2.2. *Correction for decay*

An approximate method of correcting the rapid distortion theory to take into account the effects of viscous decay has been suggested by Ribner & Tucker (1952). The method was proposed for correcting the individual components but is used here only for q^2, twice the total kinetic energy of turbulence. The decay and strain are considered to occur in alternating small steps. Each element is considered to be strained instantaneously during one step and then to decay without strain during the next. Let the subscripts D and S be used to denote the separate effects of decay and strain respectively. The decay of turbulence in the absence of strain is a function of time but may be expressed as $f_1(x_2)$, a function of the longitudinal distance x_2, as in figure 1,

$$\left(\frac{q^2}{(q^2)_A}\right)_D = f_1(x_2).$$

The value of $f_1(x_2)$ may be obtained empirically or by experiment. Similarly the effect of strain without decay may be expressed as

$$\left(\frac{q^2}{(q^2)_A}\right)_S = f_2(x_2),$$

where $f_2(x_2)$ is obtained from the rapid distortion theory. Assuming the total effect of strain and decay is given by

$$\frac{dq^2}{q^2} = \left(\frac{dq^2}{q^2}\right)_D + \left(\frac{dq^2}{q^2}\right)_S,$$

then $$q^2/(q^2)_A = f_1(x_2)f_2(x_2).$$

The method is only approximate because it assumes the flow structure is the same as that produced by rapid distortion and that the decay of the turbulence is not influenced by the flow structure.

3. Experimental arrangements

3.1. *Arrangement, laterally distorting tunnel*

A homogeneous field of pure strain in which one of the strain ratios remains unity may be produced experimentally in two ways. The first is the method used by Townsend (1954) who strained a grid-generated turbulence in a tunnel in which the cross-sectional area was kept constant, giving a zero rate of strain in the main flow direction. The walls were curved to produce equal and opposite constant rates of strain in the lateral directions. A tunnel of the same type, referred to here as a 'laterally distorting tunnel', was constructed to produce strain ratios of 6, 1 and $\frac{1}{6}$. The working section of this tunnel is shown schematically in figure 1.

For design purposes the streamlines in the distorting section were assumed to follow curves given by $x_3 = (x_3)_A e^{-cx_2}$ and $x_1 = (x_1)_A e^{cx_2}$. The reference axes are shown in figure 1, the origin is located on the centre-line at the beginning of the distorting section, c is a design constant chosen here as 0·2225/ft. and e is the base of natural logarithms. For flow following the streamlines the velocities are

$$U_2 = \text{constant}, \quad U_3 = -cU_2 x_3, \quad U_1 = cU_2 x_1.$$

The non-zero rates of strain are

$$\frac{\partial U_3}{\partial x_3} = -cU_2; \quad \frac{\partial U_1}{\partial x_1} = cU_2.$$

3.2. *Arrangement, longitudinally distorting tunnel*

The second way of producing a homogeneous strain field with one strain ratio unity is in a two-dimensional contraction. This gives flow with a positive rate of strain in the direction of the main motion and an equal negative rate of strain in one lateral direction. The rate of strain in the other lateral direction is zero. A tunnel to produce strain of this type, referred to here as a 'longitudinally distorting tunnel', was also assembled. The working section is shown schematically in figure 2.

For design purposes the streamlines in this distorting section were assumed to follow curves given by

$$x_3 = \frac{(x_3)_A}{1+bx_1}, \quad x_2 = (x_2)_A, \quad b \text{ is a design constant.}$$

The axes of reference are shown in figure 2, with the origin located on the centre-line of the tunnel at the entrance to the contraction. For the flow following the streamlines the velocities are

$$U_2 = 0, \quad U_3 = -bx_3(U_1)_A, \quad U_1 = bx_1(U_1)_A.$$

The non-zero rates of strain are

$$\frac{\partial U_3}{\partial x} = -b(U_1)_A, \quad \frac{\partial U_1}{\partial x_1} = b(U_1)_A.$$

We see that with equal longitudinal inlet velocities the two types of distorting sections give identical rates of strain when the design constants b and c are equal.

The turbulence in the tunnels was generated by two different types of grids. The first was 0·030 in. perforated steel forming a square $\frac{11}{16}$ in. mesh with bars $\frac{3}{16}$ in. wide. The second was 0·037 in. expanded aluminum forming a diamond mesh 1·2 in. × 0·60 in. on diagonals with bars 0·08 in. wide.

4. Experimental results

4.1. *Results, laterally distorting tunnel*

With the perforated metal grid in place, the velocity behind the grid was kept constant at 20 ft./s and turbulence measurements were taken along the centre-line of the tunnel. The fraction of turbulent energy in each of the three velocity

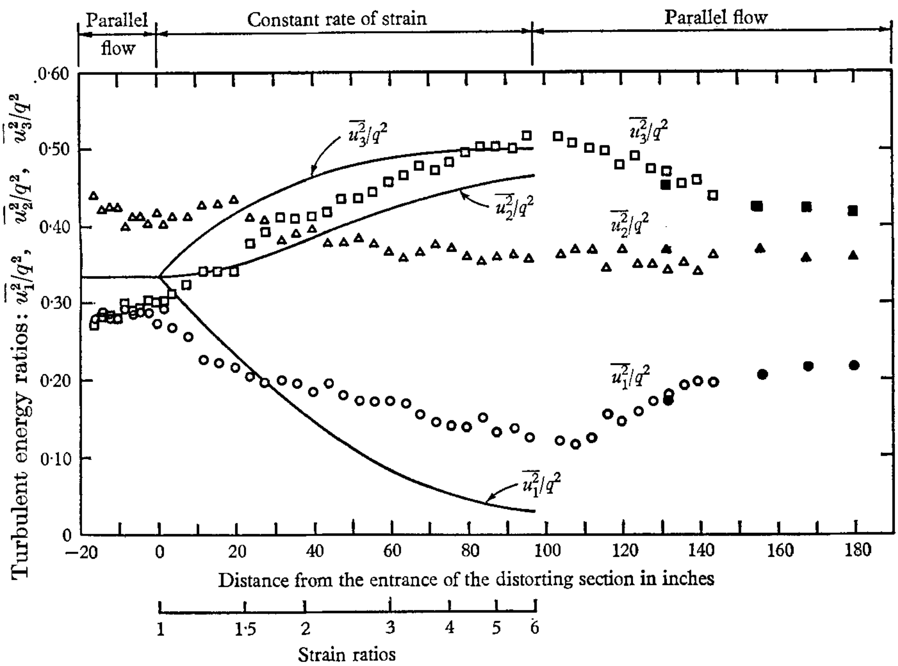

FIGURE 4. Changes in local structure parameters produced by uniform plane strain in the laterally distorting tunnel and the return towards isotropy after strain. The solid curves are from the equations of Batchelor & Proudman (1954) for rapid distortion of initially isotropic turbulence. The solid points were measured after extending the parallel duct.

components is shown in figure 4. The turbulence in the parallel section behind the grid was not isotropic, the u_2 component containing about 0·42 of the total turbulent energy compared with 0·29 for the u_1 and u_3 components (for isotropic turbulence all components would of course have values of 0·33).

We take $\overline{u_2^2}/\overline{u_1^2}$ as a measure of anisotropy in order to compare the simple grid turbulence with that found by other workers. The values found by Grant & Nisbet (1957), by Uberoi (1963) and by us are 1·39, 1·45 and 1·45, respectively.

In order to compare the experimental results with those predicted by the rapid distortion theory, curves obtained from the results of Batchelor & Proudman (1954) are shown as solid lines in figure 4. These curves are for turbulence assumed to be isotropic before strain. For the anisotropic turbulence measured behind the grid it would be possible to measure the energy spectrum and thence to produce an improved strain prediction; however, this was not attempted.

We see that during uniform strain there is a tendency for the kinetic energy of turbulence to be distributed amongst the components in a manner generally

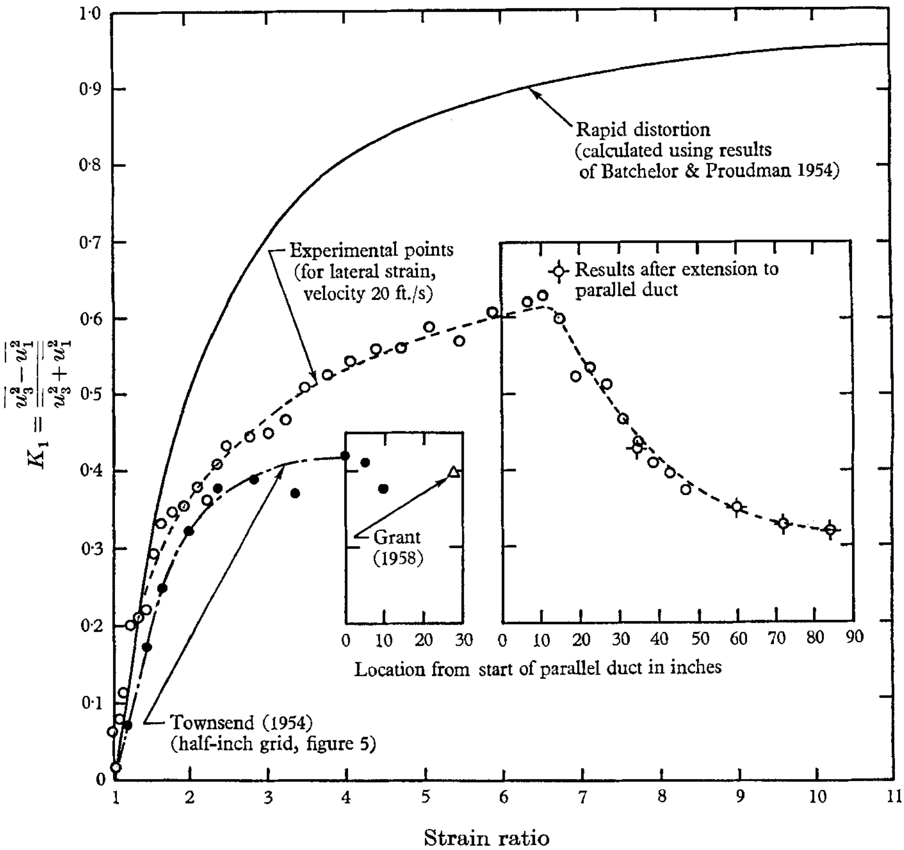

FIGURE 5. Comparison of the turbulence structure produced by uniform plane strain in the laterally distorting tunnel with that predicted by the rapid distortion theory of Batchelor & Proudman (1954) here shown as a solid line. ●, Townsend (1954, half-inch grid, figure 5); △, Grant (1958); -◇-, experimental points after extension to parallel duct; ○, experimental points for longitudinal mean velocity 20 ft./s.

similar to that predicted by the rapid distortion theory. Near the end of the distorting section, the fraction of turbulent energy in the u_1 velocity component becomes higher than that predicted, while that in the u_2 component becomes lower. It appears that energy is being transferred from the u_2 to the u_1 component, a process neglected in the theory. In the parallel flow after strain no energy is supplied to the turbulence from the mean motion, the relative energy $\overline{u_1^2}/q^2$ associated with the u_1 velocity component increases and this appears to be at the expense of energy in the high u_3 component.

To compare the results with those of Townsend (1954) and also with the rapid distortion theory, the structural parameter $K_1 = (\overline{u_3^2}-\overline{u_1^2})/(\overline{u_3^2}+\overline{u_1^2})$ is plotted against strain ratio as shown in figure 5. In all cases the experimental curves are generally similar to those obtained from the rapid distortion theory. Townsend's curve appears to reach an asymptotic value near 0·42 after a strain ratio of 4 which was the maximum obtainable in his tunnel. The results of the present experiments show that this parameter does not attain an asymptotic value, even for the strain ratio of 6, and that the asymptotic value, if it exists, is certainly greater than 0·60.† For strain ratios below 3 there is fair agreement between the present experimental results and those of Townsend, with the present results in closer agreement with the rapid distortion theory. This cannot be explained simply as the result of differences in rates of strain, since the rate of strain used by Townsend was the higher, and might therefore be expected to give results closer to those of the instantaneous distortion theory.

The falling off of Townsend's data for strain ratios near four may possibly be the result of the failure of the experimental duct to establish the hypothetical uniform straining throughout its length. Figure A1 and the corresponding measurements of Townsend (1954) show that near the entrance and exit of the distorting section the longitudinal distortion differed considerably from the hypothetical uniform value achieved in the central region. In Townsend's duct uniformity was achieved for a range of strain ratios 1·5 to 3 and in our duct for a range of strain ratios 1·5 to 4. Hence Townsends results for strain ratios 3 to 4, may be in error as may our results for strain ratios greater than four.

Figure A1 and the corresponding curve of Townsend (1954) also show that strain is not instantaneously reduced to the hypothetical zero on entrance to the parallel duct. Townsend's two measurements were taken within a region of strain. Our results show that return to isotropy took place only after the fluid had moved a short distance along the parallel duct corresponding approximately with the start of zero longitudinal strain.

In the parallel flow following the distortion the turbulence showed a marked tendency towards isotropy. In this respect the results do not agree with the conclusions reached by Townsend (1954) and the measurement of Grant (1958) whose single point is shown. Although it was not the purpose of the experiment to study the decay of anisotorpic turbulence, it was felt that additional measurements at further distances downstream were warranted. An additional length of parallel duct was added to the original tunnel and measurements extended to 84 inches from the end of the distorting duct. The results are shown in figures 4 and 5. It appears that the return to isotropy is most rapid for highly anisotropic turbulence and slower as the turbulence approaches the isotropic condition. This would partly explain the discrepancy between our results and those of Townsend and Grant, since their turbulence was closer to isotropy than ours.

4.2. *Results considering decay, laterally distorting tunnel*

There is fair agreement between experiment and the instantaneous strain theory for the structural parameters already discussed, because these parameters are

† Values of K_1 near 0·62 have recently been reported by J. Maréchal 1967, *Comptes rendus*, **265**, série A, p. 478.

fairly insensitive to decay. Agreement with the simple theory is less satisfactory when comparing quantities, such as the turbulent kinetic energies, which for practical rates of strain depend largely on decay. This is illustrated by comparing the total kinetic energy of turbulence; for a strain ratio of 6 the measured

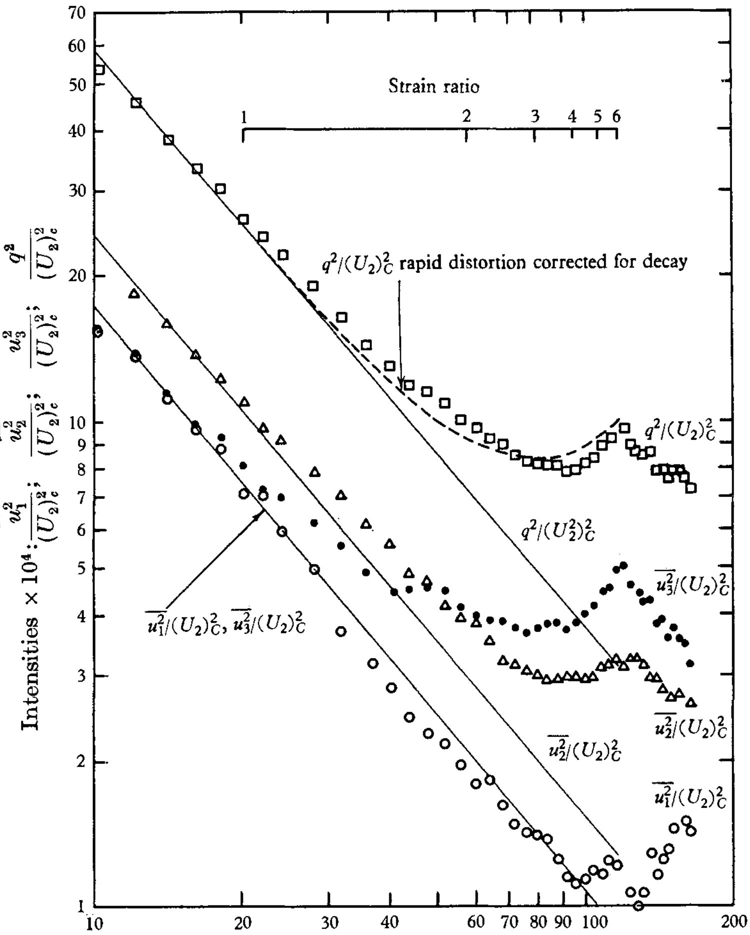

FIGURE 6. Turbulent intensities in laterally distorting tunnel and comparison of total turbulent energy with that predicted by the rapid distortion theory corrected for decay. The solid lines are for unstrained turbulence from the data of Uberoi (1963).

value of this energy decreased by a ratio of 0·37, calculated from results shown on figure 6, whereas the simple theory predicts an increase in the total turbulent energy by a ratio of 3·17 as shown in figure 3.

In order to correct the rapid distortion theory for decay, an estimate was made of the decay in the absence of strain. For turbulence behind grids in uniform flow, it is generally found that due to decay $q^2 \sim (x_2-(x_2)_0)^{-n}$ where $(x_2)_0$ is the distance to the virtual origin of the turbulence. For isotropic turbulence in the initial period, Batchelor & Townsend (1948) obtained a value of n close to unity. For slightly anisotropic turbulence, Uberoi (1963) obtained a value of $n = 1{\cdot}2$. In the present experiments the initial grid turbulence was anisotropic with a structure similar to that of Uberoi's and for this reason $n = 1{\cdot}2$ was assumed.

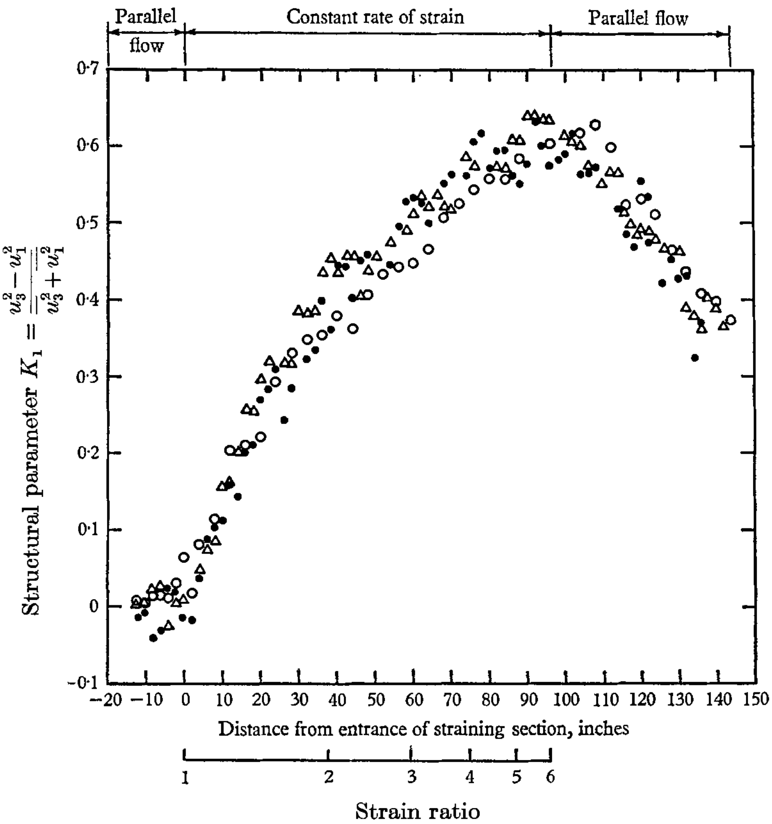

FIGURE 7. Comparison of the structural parameters K_1 in the laterally distorting tunnel for different grids and longitudinal mean velocities. ○, square perforated metal grid, velocity 20 ft./s; ●, diamond expanded metal grid, velocity 20 ft./s; △, diamond expanded metal grid, velocity 40 ft./s.

In figure 6 values of $\overline{u_1^2}/(U_2)^2_C$, $\overline{u_2^2}/(U_2)^2_C$, $\overline{u_3^2}/(U_2)^2_C$ and $q^2/(U_2)^2_C$ are plotted against distance measured from the virtual origin. The subscript C is used to denote conditions at C, located 20 in. before the distortion where the velocity is uniform as shown in figure A 1 of the appendix. Assuming (as Uberoi (1963) found) that for slightly anisotropic turbulence the fraction of turbulent energy in each of the components remains practically constant during decay, then curves would follow the lines of slope 1·2, if the turbulence were not strained. In comparison with the unstrained turbulence the actual turbulence moved in the direction indicated by the rapid distortion theory, this being less so in the case of the u_1 component. A curve of $q^2/(U_2)^2_C$ calculated using the rapid distortion theory corrected for decay shows fair agreement with experiment. For the individual components this is not true, especially in the case of the small u_1 component.

It appears that the interchange of turbulent energy between components is significant, contrary to the suggestions of Ribner & Tucker (1952).

Two sets of measurements were taken for mean duct velocities $(U_2)_C$ of 20 and 40 ft./s with the perforated grid replaced by the grid of expanded metal. The results of these experiments confirm the results already presented, as can be seen by comparing the structural parameters shown in figure 7. The structure of the turbulence also appears to be independent of the longitudinal mean velocity. This is as expected since the time scale of the distortion remains proportional to the time scale of the turbulence.

4.3. *Results, longitudinally distorting tunnel*

With the perforated metal grid in place, the velocity was held constant at 20 ft./s in the parallel flow behind the grid and measurements were taken along the

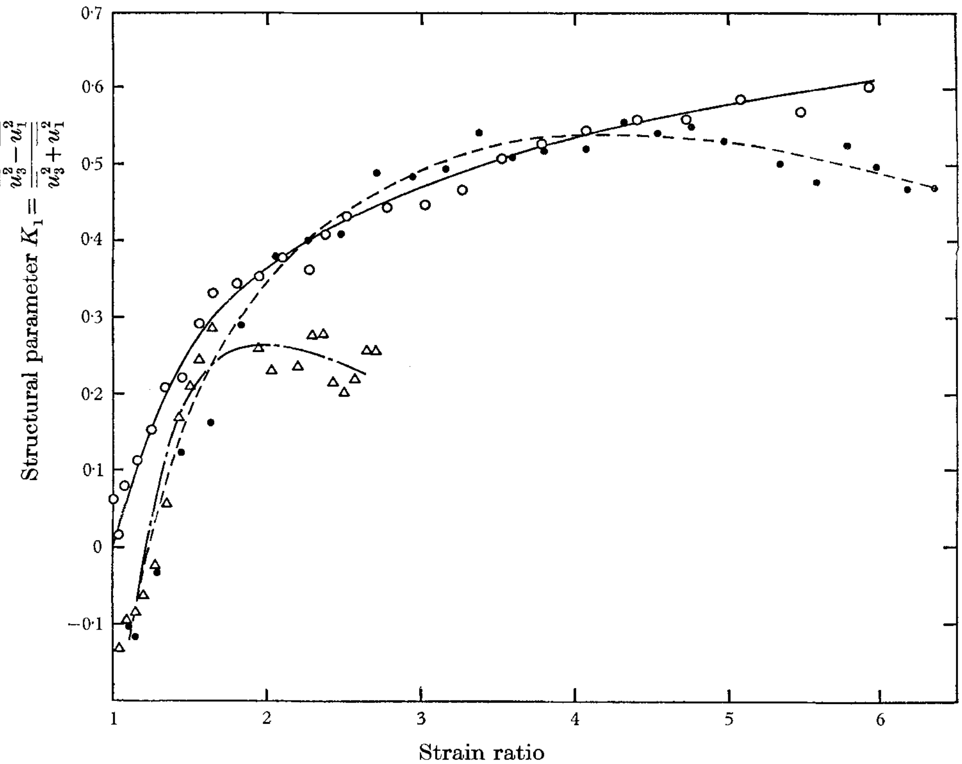

FIGURE 8. Comparison of the structural parameters K_1 in the laterally distorting tunnel with those in the longitudinally distorting tunnels. ○, laterally distorting tunnel, strain rate $\partial U_1/\partial x_1$ = 4·44 ft./s ft.; △, longitudinally distorting tunnel, strain rate $\partial U_1/\partial x_1$ = 4·85 ft./s ft.; ●, longitudinally distorting tunnel, strain rate $\partial U_1/\partial x_1$ = 14·05 ft./s ft.

centre-line of the tunnel. Two different wall adjustments were used giving strain rates $\partial U_1/\partial x_1$ = 4·85 and 14·05 ft./s ft. and strain ratios 2·85 and 6·5 respectively.

The measurements were corrected for the background turbulence of the tunnel by a method similar to that used by Uberoi (1963). The structural parameters $K_1 = (\overline{u_3^2} - \overline{u_1^2})/(\overline{u_3^2} + \overline{u_1^2})$ were plotted against strain ratios as shown in figure 8. For strain ratios up to 1·75 there is no apparent difference in the structure of the turbulence for different rates of strain. The curve for the higher rate of strain gives values of structural parameter close to 0·55 for strain ratios near 5.

The curves for both rates of strain behave similarly, showing a decrease in the structural parameter near the exit end of the tunnel. In this region the flow became unsteady making it difficult to obtain representative hot wire readings. The unsteadiness may have been caused, and the turbulence readings influenced, by conditions in the jet outside the tunnel. For this reason no conclusion regarding the structure of the turbulence can be made from the observed behaviour of the curves near the exit.

A curve for the laterally distorting tunnel is also shown for comparison. The turbulence at the beginning of strain was the same in all cases; the difference in the structural parameter for strain ratios near unity is the result of the anisotropy of the turbulence and the labelling of the reference axes with respect to strain rather than flow direction. The curves for the two different types of distortions agree remarkably well between strain ratiosof 2 and 4·5 even though the rates of strain are different.

Because of the relatively large corrections for the background turbulence of the longitudinally distorting tunnel, the results are not considered to be as reliable as those of the laterally distorting tunnel. However, it appears that the structures of the turbulence produced by the two distortions are similar and independent of small structural differences in the turbulence before strain.

5. Conclusions

The present work on the effects of uniform plane strain on turbulence is essentially an extension of the work of Townsend (1954) from strain ratios 4:1 to strain ratios 6:1. This extension has led to the following conclusions, the first three of which disagree with the conclusions of Townsend.

(i) An equilibrium structure is not developed in the turbulence for strain ratios of 4:1 and there is no evidence that an equilibrium structure is reached for strain ratios of 6:1.

(ii) The structure of the turbulence for strain ratios over 4:1 is not characterized by the anisotropy measure $K_1 = 0{\cdot}42$. Values of this parameter in the present experiments increased to 0·6 for strain ratios 6:1.

(iii) In the highly anisotropic turbulence produced by a strain ratio of 6:1 there is a strong tendency towards isotropy in the absence of mean strain.

(iv) The results indicate that the structure developed in the turbulence is mainly the result of the mean motion straining the eddies, a process covered by the rapid distortion theory, and that the total kinetic energy of turbulence may be predicted reasonably well by using the rapid distortion theory with an approximate correction for decay.

(v) It is also shown that, as would be expected, the influence of uniform strain on the structure of turbulence depends primarily on the strain and to a small extent, if at all, on the orientation of the strain relative to the main flow direction.

The experimental work was carried out in the laboratories of the Mechanical Engineering Department, McGill University and was supported by the Defence Research Board of Canada under grant no. 9551-12, held by Dr B. G. Newman.

Appendix

Laterally distorting tunnel

The tunnel used for producing uniform lateral strain was of the open-return suction type, capable of producing velocities of up to 60 ft./s. The air entered the tunnel through a two-dimensional bell-mouth, containing a curved screen

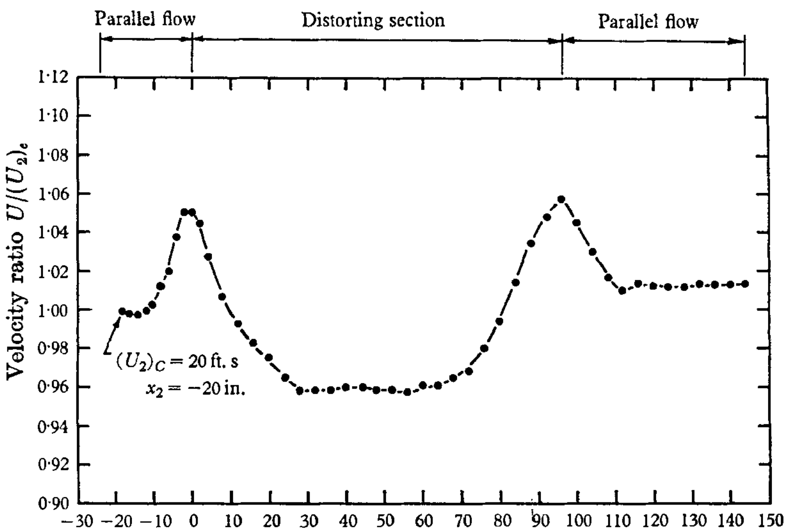

FIGURE A1. Longitudinal mean velocities along the centre-line of the laterally distorting tunnel.

and honeycomb. The arrangement of the working section is shown in figure 1. The distorting section was rectangular in cross-section and the walls followed the design streamlines except for small allowances that were made for the growth of the boundary layers. The details of this tunnel are given in a separate report, Tucker (1966).

The U_2 component of mean velocity was measured at points on the longitudinal centre-line of the duct; the distribution is shown in figure A1. Unlike the flow following the theoretical streamlines, the velocity does not remain constant but varies a total of about 9 % near the entrance and exit of the distorting section. The dimensionless velocity distribution was not influenced by the presence of the turbulence generating grid and was the same for velocites of 20 and 40 ft./s.

The turbulence was measured in the absence of the turbulent generating grid. At the beginning of distortion the longitudinal intensity was

$$\frac{(\overline{u_2^2})^{\frac{1}{2}}}{U_2} = 0{\cdot}20 \times 10^{-2}.$$

This value is low in comparison with the longitudinal intensity in the presence of the grid and correcting for the background turbulence of the tunnel by a method similar to that of Uberoi (1956) had little effect on the results.

Longitudinally distorting tunnel

For convenience an existing open circuit blower tunnel with a working section of 30 in. × 17 in. was used to generate the flow through the two-dimensional contraction; in effect the two-dimensional contraction formed a nozzle at the

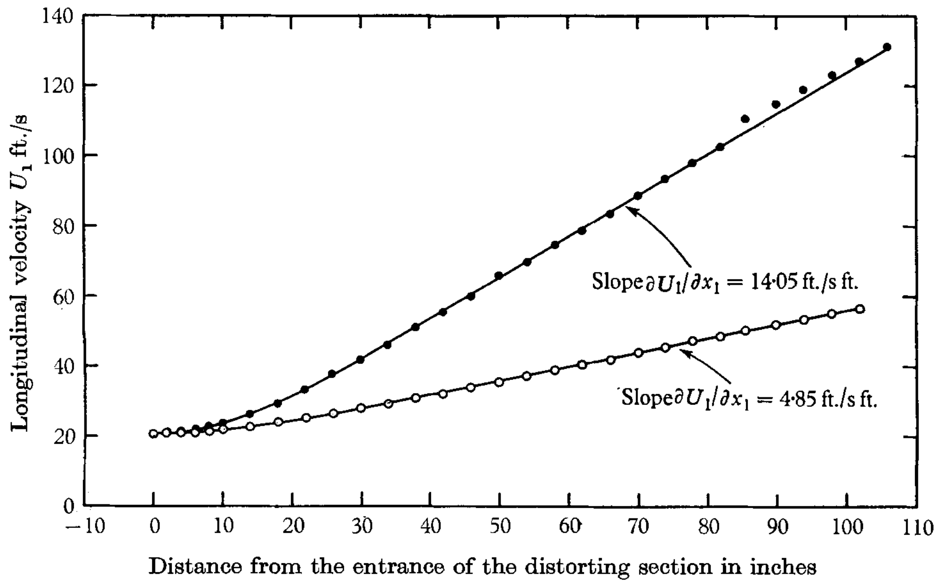

FIGURE A2. Longitudinal mean velocities along the centre-line of the longitudinally distorting tunnels.

exhaust end of the tunnel. Although the tunnel contained three screens and a honeycomb in the settling chamber the background turbulence was relatively high. The longidutinal intensity of the turbulence at the beginning of the distorting section in the absence of the turbulence generating grid was

$$\frac{(\overline{u_1^2})^{\frac{1}{2}}}{U_1} = 0{\cdot}55 \times 10^{-2}.$$

This value was not small in comparison with the longitudinal component of the intensity with the grid present. An effort to reduce the turbulence by placing an additional screen in the settling chamber of the tunnel was not effective and it was necessary to correct the measurements for background turbulence, by the method of Uberoi (1956).

The arrangement of the working section is shown in figure 2. The contraction consisted of two parallel strips of plexiglass held between two horizontal plane surfaces by through bolts. The plexiglass strips forming the side walls of the tunnel could be adjusted to give different rates of strain. There was no parallel settling length after the contraction.

For the two adjustments of the tunnel walls the velocities measured along the tunnel centre-line are shown in figure A2. The strain rates remained constant throughout the distorting section, except for a small region near the entrance and in this respect two-dimensional contraction appeared to be superior to the lateral type of distorting section.

Turbulence measurements

The turbulence intensities were measured using a commerically available Disa constant temperature hot wire anemometer 55A01. All wires were platinum plated tungsten 0·005 mm diameter approximately 1 mm long. To obtain the

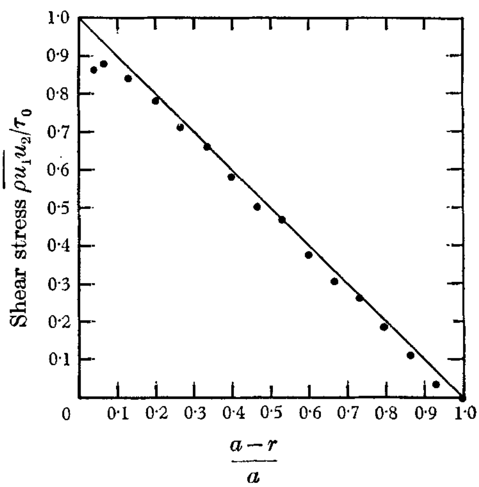

FIGURE A3. Comparison of the shear stresses measured by hot wire with the theoretical linear distribution. Reynold's number based on centre-line velocity and diameter is $1{\cdot}08 \times 10^5$. u_1 and u_2 are the fluctuating velocities in the longitudinal and lateral directions respectively. $\overline{u^2u^2}$ was measured by hot wire. τ_0 is the shear stress at the wall from measured pressure drop, ρ is the density, a is the inside radius of the pipe and r the radial distance from the pipe centre.

longitudinal component of turbulence, a single normal wire was positioned perpendicular to the main flow direction. The equation, $E^2 = A + BU_2^C$ was assumed and the values of the constant A, B and C determined by computer from calibration values of bridge voltage E and longitudinal velocity U_2 in a low turbulence air stream. The equation for the fluctuating velocity was obtained from the equation assuming low turbulence.

To determine one of the lateral components of turbulence, say the u_3 component, a single 45° slanting wire was placed in two positions in the u_3 longitudinal plane at angles of θ and $-\theta$ to the main flow direction obtained by rotating the probe about its axes. An equation of the type $E^2 = A + B(U_2 \sin\theta)^C$ was assumed. As before the constants were determined by calibration and the equation for the fluctuating velocity obtained assuming low turbulence. The angle of the slanting wire was found by rotating the probe about its axis in a slide projector and measuring the maximum angle projected on a paper screen. Repeated measurements of the same angle varied less than 0·2°.

The equation for the longitudinal components given above, assumes that the heat loss from the wire depends only on the component of velocity normal to the wire. Because this assumption is not strictly true, it was considered necessary to check the hot wire method by measuring the shear stress distribution in fully developed turbulent pipe flow and comparing with the expected linear distribution obtained from the pipe pressure drop. The comparison is shown in figure A 3. The hot wire method appears to be satisfactory since the hot wire measurements shown as points fall close to the pressure drop straight line.

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